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The Critical Thinking Dispositions of Prospective Mathematics Teachers at a South African University: New Directions for Teacher Training

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ABSTRACT The researchers report on findings obtained by means of a sequential, explanatory, mixed-method study with a convenient and purposefully selected group of second-year Mathematics students at a South African university (n = 29). Using a self-developed questionnaire, the researchers profiled how important students perceive critical thinking dispositions to be in Mathematics. Further, narratives focused on understanding the factors that may influence these perceptions.The findings revealed that dimensions of inquisitiveness, judiciousness,open-mindedness, ***Systematicity***, truth seeking, analyticity and confidence appear to require encouragement. In particular, the factors that influenced the perceptions of the students centred on classroom-related issues.

INTRODUCTION

A changing and challenging world demands not only the teaching of facts, but also the cultivation of critical thinking, including the development of critical thinking skills, as well as dispositions at all levels of education (Qing et al. 2010). The new Curriculum and Assessment Policy Statement (CAPS), which guides yet another reform in South African education, aims to equip students for the changes and challenges of the 21st century. Students should be able to "use mathematical process skills to identify, investigate and solve problems creatively and critically" (Department of Basic Education 2011). Although, this is a noble goal, an average of only 13 percent and 14 per cent was obtained by Grade 9 learners in the Annual National Assessments (ANA) for Mathematics in 2012, and 2013, respectively (Fredericks 2012; Department of Basic Education 2013), which indicates that the development of the mentioned process skills appears to be problematic (Leong et al. 2013; Gunhan, 2014; Tsanwani et al. 2014). Similar trends in Mathematics performance are noted internationally. Canada has dropped out of the top ten in international math education standings (Alphonso 2013). In comparison to the 28 percent of Taiwanese and about 20 percent of students in Hong Kong, Korea and Finland who achieved an advanced level in Mathematics, only 6 percent of high school students graduated in the United States during 2009 at an advanced level in Mathematics (Kilpatrick 2011).

Palmer (2007) and Paul and Elder (2008) consider critical thinking to be a prerequisite for critical thinking dispositions, while Facione (2000) and Profetto-McGrath (2003) highlight the importance of dispositions to improve learning in Mathematics. Knowing Mathematics involves more than just memorizing facts, it also involves dispositions to explain convincingly one's ideas to others (Francisco 2013). The researchers support the view of Profetto-McGrath (2003), and therefore state that a lack of critical thinking dispositions may contribute to the perturbing results in Mathematics nationally and internationally. Eli and Mohr-Schroeder (n. d.) continue to emphasize the importance of building habits of mind in Mathematics as they promote a broad foundation for mathematical knowledge and are essential for problem-solving.

Research and working group discussions determined that dispositions among Science and Mathematics students at school level are problematic (Cuoco et al. 1996; Saleh and Khine 2011; Elyousif and Abdelhamied 2013; Matsuura et al. 2013; Temel 2014). Curriculum development efforts attempt to provide students with skills to successfully practice thinking dispositions. In

this regard Francisco (2013) and Temel (2014) advocated for student-centred approaches to teaching Mathematics, where collaborative work, discovery learning, discussions, problem-solving and construction of knowledge are used to shape students' views on learning and thinking and enhance achievement in Mathematics (Cuoco et al. 1996; National Centre for Education Evaluation and Regional Assistance 2013); however, teachers' understanding of these dispositions is problematic and they lack the experience and skills to develop them (Elyousif and Abdelhamied 2013). Three international quantitative studies determined the critical thinking dispositions of pre-service teachers at a private university (Facione et al. 1995), among community college student teachers (Bers et al. 1996), as well as pre-service physical education teachers at nine universities in the USA (McBride et al. 2002). Among others, truth seeking appears to be a disposition that could be more positively endorsed by student teachers.

To the best of the researchers' knowledge, no national or international research on critical thinking dispositions among pre-service Mathematics teachers has been conducted. The current paper extends the international research base on critical thinking dispositions, as it is a first attempt at creating a profile of the characteristics of critical thinking dispositions among pre-service Mathematics teachers at a South African university, and at establishing the factors that may influence the importance with which critical thinking dispositions are viewed. The findings of this paper can be compared to the findings of the international studies, according to which tendencies in the dispositional development of pre-service teachers in general can be identified. The identification of these tendencies could lead to suggestions for enhancing teacher-training curricula nationally and internationally.

In order to emphasize the important role of critical thinking dispositions in Mathematics, the following section elucidates this relationship.

The Importance of Critical Thinking Dispositions in Mathematics

Mathematics has mainly been regarded as a set of rules that has to be mastered, comprising arithmetic operations, mysterious algebraic equations and geometric evidence (Van de Walle 2004). The "follow-the-rules", calculation-dominated and response-oriented view is a distortion of what Mathematics really is (Van de Walle 2004). On the contrary, students should ask questions and evaluate their own answers (Van de Walle 2004), which implies the application of critical thinking skills. A certain disposition is therefore necessary to recognize when critical thinking skills are needed and there must be a willing mental effort to apply the skills (Facione 2000, 2011). In support of Facione (2000) and Profetto-McGrath (2003), the present paper elucidates that such dispositions are crucial for critical thinking and that critical thinking will not take place or will be inferior if these dispositions are not regarded as important. According to Anderson (2010), these dispositions are tools that are employed skilfully and mindfully by people when confronted with problems, the solutions to which are not immediately apparent.

Thinking dispositions develop in classroom environments where students have opportunities to reason and construct their own knowledge (Staples 2007; Van de Walle et al. 2010). In order to learn Mathematics, students must learn to think critically through problem-solving (M·rcuþ 2005). They should be able to communicate and reason mathematically, and develop confidence in their ability to use Mathematics (Heddens et al. 2009). For this purpose they require the dispositions of having confidence in reasoning, being committed to solving challenging problems (Facione 2011), and seeking the truth by probing and searching for the best possible solution to a problem (Facione and Facione 2010; Facione 2011).

Important critical thinking dispositions in Mathematics include an open mind and appreciation of new ideas, scepticism, searching for evidence and logic, the consideration of alternatives, perseverance, a tendency to reflect on learning, the creative use of imagination and curiosity, and integrity, diligence and fairness (Grotzer n. d. ; NCTM 1989). Open-mindedness calls for respect of others' opinions (Carroll 2004). Being alert to alternatives and the consequences of decisions (Facione 2011), as well as being attentive to reasoning (Ricketts et al. 2003), are tenets of effective critical thinking dispositions in Mathematics.

Inquisitiveness or intellectual curiosity (Facione 2010, 2011) is required to obtain the background knowledge for solving basic problems

(M·rcuþ 2005; Winch 2006; Sezer 2008) for which a systematic approach is needed (Consiglio 2003; Department of Education 2003; Ayalon and Even 2008; Fradkin et al. 2010; Van de Walle et al. 2010). Problem-solving in Mathematics requires focused and persistent ways of working (Ricketts et al. 2003), distinguishing between crucial and unimportant facts (Facione 2011), as well as reflecting on the decisions made. In this regard, judiciousness as a disposition drives the evaluation of decisions to solve problems (Facione 2011). The researchers explored that if students do not work systematically, are not open-minded and do not seek the truth, it will be difficult for them to apply the higher-order thinking skills required in Mathematics (such as gathering, analyzing, organizing and evaluating information).

Cuoco et al. (1996) and Maree et al. (2004) make it clear that dispositions are more important than Mathematics results. Cobb and Hodge, as well as Ames and Archer (cited by Gresalfi and Cobb 2006), agree that positive critical thinking dispositions serve as a motivation for learning and that students are intensively and more effectively involved in Mathematics activities when critical thinking dispositions are present. There are, however, factors that might restrict the development of students' critical thinking dispositions.

Factors Affecting Critical Thinking Dispositions

The cultural environment and cultural differences could contribute to poor thinking dispositions (Nisbett et al. 2001; Nisbett and Norenzayan 2002). Individuals from a Western culture who grow up in conditions where the focus is on personal freedom, choice, curiosity and diversity will most likely show characteristics of analyticity (Nisbett et al. 2001). *Emotional factors* can also affect critical thinking dispositions (Carroll 2004). If a person fears that he or she will not succeed in solving a problem, he or she probably will not even try to do so (Carroll 2004). Tension, nervousness, withdrawal, negative feedback, fear and previous negative or positive experiences in the teaching and learning situation can impact on the manifestation of critical thinking dispositions, such as having or lacking self-confidence in reasoning (Costa and Kallick 2009b).

A teacher, who acts as a mediator, who intentionally selects and organizes the stimuli for the students (Fraser 2006), enables them to distinguish between relevant and irrelevant information, helps them to solve problems with confidence, and contributes to their students' dispositions of working efficiently and systematically (Fraser 2006; Costa and Kallick 2009c).

Questioning can encourage the development of critical thinking dispositions (Elder and Paul 2004; Woolfolk 2004; Costa and Kallick 2009b). When students are exposed to higher-order questioning, critical thinking dispositions, such as curiosity, which encourage the construction of new ideas, are aroused.

Students who rely on *memorizing content* do not develop skills to gather and use information (Grabe and Grabe 2004). They must be taught to generalize and analyze effectively to solve complex problems and draw valid conclusions (English 2008; Weimer 2010; Hanford 2011; Johnson 2012).When students are taught to analyze and generalize when solving complex problems, as well as to draw conclusions, a sense of competence and confidence in their own critical thinking is encouraged.

The use of *reflection* during teaching is important to encourage the development of critical thinking dispositions. If students do not learn to develop the ability to reflect on ideas, they will find it difficult to think about a problem or to apply their ideas in other situations (Costa and Kallick 2009c).

The classroom should contain a variety of *data sources* such as books, encyclopaedias, databases, calendars and dictionaries that serve as sources from which students can collect information to encourage the critical thinking dispositions of curiosity and truth seeking (Costa and Kallick 2009b).

Objectives

The objective of the paper is to determine the perceptions of prospective Mathematics teachers regarding the different critical thinking dispositions.

METHODOLOGY

Framed within a pragmatic research paradigm, explanatory, sequential, mixed-method research was employed to collect data. A quanti-

tative, descriptive survey research strategy with a self-developed closed questionnaire comprising 46 semantic scale items was employed as a data-collection instrument. The researchers wanted to determine objectively how important the participants considered critical thinking dispositions in Mathematics. The participants had to rate the importance of the critical thinking dispositions on a seven-point semantic scale that ranged from 1 (important) to 7 (unimportant). A number of questions related to different dimensions of each of the seven critical thinking dispositions as identified by Facione (2000) were posed.

Qualitative non-interactive phenomenological research was undertaken by means of the writing of narratives (Bold 2012). The results were used to better understand the factors and experiences that may influence the development of the students' critical thinking dispositions. The narratives required an individual explanation from each student regarding their rating as important or unimportant in the questionnaire of the factors that influence their perceptions of the different critical thinking dispositions.

In order to ensure validity and reliability, the researchers adhered to the following procedures as explained below.

Reliability and Validity of Instruments

The reliability of the questionnaire was determined using a pilot study with a group of second-year Mathematics education students for 2010. Since the reliability of the first and second pilot studies was low, some questions were omitted or altered and new questions were formulated. After a third pilot study, acceptable Cronbach alpha coefficients (which varied between 0. 596 and 0. 871) were noted for all questionnaire items.

Credibility of the qualitative research was assured by the empanelment of a colleague who is an expert in the field of critical thinking dispositions to verify the questions in the questionnaire, as well as the researchers' interpretation and analysis of the data. Further, detailed and rich descriptions of the students' experiences were obtained about the factors that influenced their view regarding the importance of their critical thinking dispositions.

Sample

The population for this study comprised all education students in South Africa majoring in Mathematics. It was impossible to involve the entire student population in the study, and therefore a study population that consisted of education students (n = 29) majoring in Mathematics at a university in one of the nine provinces, Gauteng, was chosen. Non-probability, convenient, purposeful sampling was used, because the students were taught by the researchers and therefore easily accessible. As lecturers, the researchers became aware of second-year education students' lack of critical thinking dispositions, hence the purposeful focus on second-year students.

Data Analysis

Frequencies were calculated for the responses obtained on the seven-point scale for each of the questionnaire items. It was important to the researchers that the students consider all the different dimensions related to each of the critical thinking dispositions as important (1) and indispensable for Mathematics. Dimensions of the critical thinking dispositions to which the minority and majority of the participants assigned a value of 1 guided the formulation of the questions for the narratives to understand what factors influenced the students to regard certain dimensions of the critical thinking dispositions more or less important than other dimensions.

Deductive and inductive content analyses of the factors that influenced the participants' perceptions of critical thinking dispositions were used to extract the themes from the narratives (Nieuwenhuis 2007) according to the following categories: person, incidents, experiences and the use of language and meaning attached to events, as proposed by the literature for a narrative analysis (McMillan and Schumacher 2006).

Ethical Issues

Ethical aspects related to the research problem, objectives and research questions, data collection, data analysis and interpretation, and the reporting and dissemination of the research were taken into consideration (Creswell 2009). The research aim and the students' voluntary involvement in the research were discussed with

the students, after which written consent was obtained. A code was allocated to each student to ensure confidentiality and anonymity.

RESULTS

Quantitative Data Analysis

It is disconcerting to note that none of the dimensions related to the various critical thinking dispositions were regarded as very important by any of the students. A response of 1 (important) was considered as ideal because the literature confirms that students should consider all critical thinking dispositions as very important in order to apply critical thinking skills successfully in Mathematics (Facione 1990; Bailin et al. 1999). Certain dimensions within the various dispositions appeared to be regarded as less important than others. In Table 1 a descriptive analysis of the data is reported. For the purpose of the paper, the researchers firstly draw attention to problematic dimensions in each of the dispositions that possibly require encouragement. In addition, the dimensions which the students appeared to value are also highlighted, as these dimensions need to be reinforced so that students stay committed and internally attuned to keep on applying them.

Truth Seeking

Seeking the truth is concerned with finding the best possible solution to a problem and understanding an issue thoroughly before making a decision regarding the best way to solve a problem.

Table 1: Frequencies for dispositions

Semantic scale / *Question*	*1* *f*	*2* *f*	*3* *f*	*4* *f*	*5* *f*	*6* *f*	*7* *f*	*n*
Truth Seeking								
2:	…understand a Maths problem completely before making a decision							
	17	7	3	1	1			29
5:	…not allow prejudice to influence decisions							
	8	10	8	3				29
Open-mindedness								
11:	…show understanding for the worldviews of others							
	6	8	9	2	3		1	29
12:	. . . show respect towards people whose ideas differ from mine							
	16	11		2				29
Analyticity								
14:	… my decisions should add meaning to my life							
	18	6	3	1			1	29
15:	…foresee short-term outcomes for decisions							
	6	13	6	3	1			29
Systematicity								
21:	…plan thoroughly before tackling a Maths assignment							
	6	10	4	7	2			29
23:	…set goals for myself according to which I tackle my studies							
	14	6	6	2	1			29
Confidence in Reasoning								
25:	…reflect on decisions that impact on my future							
	18	9	2					29
30:	…reflect on (think about) my decisions							
	7	14	5	2		1		29
Inquisitiveness								
40:…study new information that has no relevance to my studies								
	2	3	6	8	3	3	4	29
43:…work in advance and come to the Maths class prepared								
	13	5	5	2	3	1		29
Judiciousness								
46:… when I am in doubt, I use other sources to verify my facts								
	16	7	5		1			29
50:… acknowledge that I am wrong								
	3	9	11	5	1			29

Although in the majority, only 17 participants (58. 6%) viewed it as important to understand a Mathematics problem fully before making a decision, while only eight participants (27. 5%) considered it important not to let prejudice affect their decisions.

Mathematics relies on deductive evidence and not on discovery and experiments (Ben-Hur 2006). Two important factors in the teaching of Mathematics are procedural and conceptual knowledge and the interaction between the two (Bennie 2005; Ben-Hur 2006). Therefore students should be able to give reasons for statements in order to verify the truth. Halmos (quoted by Schoenfeld 1992) claims that Mathematics focuses on problems and problem-solving, for which the critical thinking disposition to seek the truth is inseparable from the problem-solving process. When students are unplanned, impulsive and unsystematic in solving problems, they may not consider it important to identify reasons for the steps in problem-solving and the quality of their knowledge would possibly be inadequate (Feuerstein 2007; O'Daffer et al. 2008; Eggen and Kauchak 2010). It would thus appear that the majority of the participants acted impulsively and did not have all the facts to hand before making a decision.

The researchers argue that a lack of knowledge or vague and cursory observation can contribute to the reason why students are not able to explain why a certain mathematical step is followed when solving a problem (Feuerstein et al. 2002; Feuerstein 2007). Students who are not able to identify reasons for solving problems in a specific way may also not be aware of the consequences of their actions. From the data it seems that many students do not consider truth seeking very important, as there were many students who did not allocate a 1 on the semantic scale to the dimensions related to this disposition.

Open-mindedness

Only six (20. 6%) participants considered it important to understand the worldview of others. Sixteen participants (55. 1%) felt it was important to show respect toward someone whose ideas differ from theirs.

Being open to a variety of ideas has the potential to improve learning (Eggen and Kauchak 2010). The question arises as to whether and how prospective teachers will develop this critical thinking disposition in their students against the apparent unimportance with which open-mindedness is viewed. The critical and creative identification and solving of problems may not be successful when a person is not receptive to the ideas of others. The challenge in a Mathematics class should be to seek alternative strategies to solve problems (Krulik and Rudnick 1999; Paul and Elder 2008), and therefore an open mind, respect, patience and tolerance toward others are essential.

Analyticity

The majority, 18 participants (62%) in all, apparently considered it important that their decisions should add meaning to their lives, while 6 participants (20. 6%) claimed that it is important to be aware of the short-term outcomes of decisions. To be aware of consequences means to be aware of potential problems and to note the foreseeable consequences of outcomes (Facione 2011).

Inductive and deductive reasoning, as well as the critical thinking dispositions needed to analyze, synthesize and evaluate, are used in Mathematics (Borich 2004; Ayalon and Even 2008). To use these skills successfully, it is essential that students should be aware of reasons and facts when problems are solved (Facione 2010), as well as of the consequences of their decisions. Mistakes made in Mathematics should serve as opportunities from which they can learn (Hiebert cited by Van deWalle et al. 2010). Prospective teachers must have the critical thinking disposition to be aware of consequences, so that they may be able to explain to their students the implications of the application of an incorrect decision in respect of problem-solving. Based on the responses, it could be argued that there are many participants who do not regard the disposition as very important (1) and therefore might need encouragement in developing the propensity for working analytically.

Systematicity

Six participants (20. 6%) apparently planned carefully before they tackled a Mathematics assignment, while only 14 participants (48. 2%) probably set goals for themselves according to which they tackled their studies.

Since a systematic approach in Mathematics is essential (Ayalonand Even 2008; Fradkinet al. 2010), the responses indicate that the apparent unimportance with which this disposition is perceived could attribute to poor performance in Mathematics.

Mathematics involves the systematic generating and applying of strategies for problem-solving, and deciding whether they lead to sensible solutions to problems (Ricketts et al. 2003; Van deWalle et al. 2010; Facione 2011). A systematic approach is an important critical thinking disposition in Mathematics (Department of Education 2003; Jacobs 2004), and students who consider a systematic approach to be unimportant might not persist and might not be focused, orderly and diligent in the way they solve problems (Ricketts et al. 2003).

Strategies that can be used for a systematic approach include having a problem-solving plan (Woolfolk 2004; Tileston 2005; O'Daffer et al. 2008; Eggen and Kauchak 2010), analysing problems in the finest detail and distinguishing between relevant and irrelevant information (Facione 2011). It seems reasonable to conclude that many of the participants might not have strategies available for a systematic approach to problem-solving, or that they might not be capable of and dedicated to applying strategies, because many of them did not view this disposition as very important (1).

Confidence in Reasoning

Confidence in reasoning needs encouragement, as only seven participants (24. 1%) considered it important to reflect on their decisions and 18 (62%) considered it important to reflect on decisions that impact on their future.

Students must develop self-confidence, perseverance and the courage to logically and critically solve problems (Van de Walle et al. 2010). In addition, they need to think about the answers to problems, ensure that sufficient information for solving problems is available and reflect on their answers as important steps in the problem-solving process (Woolfolk 2004; Tileston 2005; O'Daffer et al. 2008; Eggen and Kauchak 2010). These steps can encourage self-confidence in reasoning as they encourage systematic thinking and problem-solving. The responses are worrying as they indicate that students might lack self-confidence. The researchers argue that a lack of confidence could be one reason why students give up so easily, do not complete Grade 12 or do not succeed at university. If pre-service Mathematics teachers are not confident in reasoning, they may also fail to cultivate confidence in their learners. Lacking confidence in reasoning could also be attributed to limited availability of strategies to solve problems.

Inquisitiveness

To be inquisitive means to continue to be well informed and implies intellectual curiosity (Costa 2009; Costa and Kallick 2009a; Facione 2011). The value of inquisitiveness in Mathematics implies, among other things, that students seek alternative solution methods (Elder and Paul 2002). Only 13 participants (44. 8%) indicated that they worked ahead to prepare themselves for the Mathematics class, and two participants (6. 9%) indicated that they consulted and studied new information.

The responses are alarming, especially when seen against the background of the knowledge explosion in technological fields, where inquisitiveness is central to teaching and learning (Eggen and Kauchak 2004; Facione 2010).

Further, a lack of curiosity may result in students not being receptive to others' ideas, acting impulsively and not persevering.

Judiciousness

A small number of participants (16, 55. 1%) indicated that they regarded it as important to recognize when they were wrong, where as only three participants (10. 3%) indicated that it was important to change their views on a matter.

Students must have an understanding of other methods for problem-solving and recognize that they have to adopt a variety of methods that lead to a solution (Hiebert cited by Van de Walle et al. 2010). Since the students appeared not to consider this critical thinking disposition to be important, it appears that their responses to this question do not support the statements in the literature which argue that problem-solving should be accompanied by investigating diverse problem-solving methods (O'Daffer et al. 2008).

To have well-considered judgement means, among other things, that students will realize

that problems can have more than one solution (Facione 2011). It would appear that the participants did not realize the meaning and value of judiciousness, as only a few of them indicated that they were able to recognize when they were wrong, and were able to change their views on an issue.

Qualitative Data Analysis

With regard to the category person, parents, teachers and peers, the personal attitude of the participants themselves, and teaching and learning experiences, as well as realising the positive meaning that the dispositions have for living were consistently mentioned by most participants as factors that contributed to their regarding critical thinking dispositions as important:

Both groups, namely those who considered critical thinking dispositions to be unimportant, and those who considered them to be important, mentioned the significant role of the person himself or herself: "*my personal best in what I set my mind to and what I find interesting", "dedication within myself in getting to enjoy Maths problems"* and *"to see or prove that I can do rough problems"*. Moreover, educators, parents, family and friends were also regarded as important in the development of critical thinking dispositions, as evidenced in the following words of the participants*: "my lecturer always says that if you are given a problem you should ask yourself questions like what am I given, what can I do with it, what do I know about it, and write all the steps or the possibilities to tackle or solve the problem*", *"[my lecturer] taught us steps on approaching assignments*". Parents provide guidance by encouraging students to do the following: "*before making decisions you should think critically about your decisions*". Friends contribute by" *motivate[ing] me to acknowledge that I'm wrong"*.

In addition, the responsibility to be curious, open to ideas, thoughtful in judgement and aware of consequences, to have a desire to improve oneself, and to possess endurance, inner motivation and a positive attitude towards Mathematics, were mentioned as factors that could promote the development of critical thinking dispositions: *"it makes a person more open-minded and give you respect toward each individual ideas or reasoning". "it will bring you closer to your long-term goals or outcomes", "to add meaning to personal life and what [he] can provide to the community,"to work on a [Mathematics] problem till the end", "contribute to my future and life" and "gives you confidence and positive attitude when approaching a problem"*.

Emotional factors mentioned by the group of participants who considered critical thinking dispositions to be unimportant, which seemingly have a negative effect on such development, were mainly negative comments from peers, own stubbornness, shyness and low self-esteem. Some of the responses were "*[I am] embarrassed to ask a question*", "*I feel like a looser if I got maths problem[s] wrong*" and friends who "*[raise] lousy points*". The group of participants who considered critical thinking dispositions important highlighted positive self-esteem and a sense of self-satisfaction as essential for developing dispositions: "*[dispositions]boost your confidence in solving maths problems", "[you] find a way of dealing with your studies and obtain better marks"*, and "*[dispositions] give you confidence and a positive attitude when approaching a problem"*.

The participants who regarded critical thinking dispositions as unimportant noted that they lacked time and therefore did not plan. On the other hand, participants who considered critical thinking dispositions important used time to understand problems and persevered to reach an answer. For both groups time thus played a role in the application of critical thinking dispositions. According to the participants, dispositions to critical thinking will develop when they "*start caring more about how to get to the answer*","*look back to check your mistakes"*, spend more time "*to understand the problem*" and to *"look at what [you] have and where [you are] heading*", showing "*more interest in the work*", and "*slow[ing] down"* when solving problems, to *"first gain perspective*" and base decisions to solve problems on "*facts and not on feelings"*, and constantly *"check if you [are]on the right track*".

Insufficient skills and giving up too easily when the work is regarded as too hard and challenging are factors that contribute to the view of critical thinking dispositions as being unimportant: *"me just being careless and ignorant", "[I] don't plan ahead", "I should start caring more about how to get an answer"* and "*before*

making decisions you should think critically about your decisions".

With regard to meaning, life and problem-solving were viewed as meaningless by the group that considered critical thinking dispositions less important, while the group that considered them important mentioned endurance, a vision of the future, performance, success and applying knowledge as factors that contribute to viewing critical thinking dispositions as unimportant or important. Some of the responses were: "*[I continue] until I get the right answer*", "*I want to know why, so I can understand how*", *"to prepare myself for the work I have to do"*, *"[to solve problems]in real-life situations*" *"[to avoid]meaningless decisions", "If I know the reason, I'll know how to solve the problem"*. In addition, it was noted that *"taking time to sort out the important info in the beginning saves you time in the end*", "*we can't go through life only thinking about ourselves. We have to think of others too*", *"[it]will bring you closer to your long-term goals or outcomes"* and *"[make you] a better person"*.

In respect of incidents and experiences, both groups mentioned that learning experiences are crucial for valuing critical thinking dispositions. The participants who considered critical thinking dispositions to be unimportant blamed learning experiences that lack opportunities for reasoning and modelling of critical thinking dispositions, hard work and a lack of questions, little or no real-life applications of knowledge, uninteresting work and a lack of resources to complete tasks as contributing factors. Some of the responses were: "*questions [that] are mostly beyond my competence*" and "*enough reasoning was not involved*". Participants mentioned that the teacher's application of problem-solving strategies encourages open-mindedness and inquisitiveness. Further, the dedication of teachers encourages learners to apply thinking dispositions, the availability of resources stimulates thinking, and feedback on students' answers motivates the application of critical thinking dispositions. In addition, the application of knowledge to real-life situations and problems, and teachers who recognize students' individual ways of knowledge construction are factors that influence the importance with which critical thinking dispositions are viewed*: "[teachers] assist me to think better", "educators' commitment", "positive feedback in solving problems in Maths"*, teachers should use *"different methods to tackle one topic*", *"taught us steps in approaching assignments"* and teachers guide the execution of tasks by *"[giving] assessment criteria"*.

The narratives of participants who considered critical thinking dispositions unimportant revealed, in some cases, a *negative tone and were limited in description regarding what contributes to the importance with which critical thinking dispositions are viewed*, in comparison with participants who considered critical thinking dispositions important.

In summary, the participants who regarded critical thinking dispositions as unimportant appeared to lack exposure to learning experiences that encouraged the development of critical thinking dispositions. This lack of exposure could possibly have had a negative effect on their performance and self-esteem, and could consequently have contributed to a negative vision for the future and experiencing existence as meaningless.

Integration of Quantitative and Qualitative Data

The quantitative data indicated that all the critical thinking dispositions may need encouragement as none of the dimensions for each of the dispositions was regarded as very important by any of the participants. The researchers conclude that the participants might have had a narrow understanding of the meaning and value of the critical thinking dispositions and limited strategies to apply the dispositions. The aforementioned could have contributed to their not being internally attuned and committed to applying the dispositions for effective critical thinking.

The factors cited by the participants that influenced them to regard the application of critical thinking dispositions as important link with findings in the literature that the factors mentioned in relation to affecting the establishment and influence of critical thinking. These factors include peers, teachers and parents, as well as the classroom situation and teaching strategies (Elder 2007; Costa and Kallick 2009b). The participants cited a number of additional factors that prompt the application of critical thinking dispositions which could not be located in the literature. These factors include a lack of time during teaching to model critical thinking dispositions, students' over-reliance on others pro-

vide answers, being afraid of disappointing teachers and parents and not living up to their expectations. In addition, refraining from voicing an opinion, a lack of enriching life experiences and prior learning, not having a vision for the future, and work that is not challenging and meaningful appear to play a role in prompting the application of critical thinking dispositions.

The responses of the participants who regarded critical thinking dispositions as unimportant cited personal attitudes, personal and emotional factors, a lack of time (which contributes to poor planning) and personal inability as factors that impede the application of critical thinking dispositions. If teachers do not model critical thinking dispositions, and provide limited opportunity for meaningful, real-life application of knowledge, the importance with which critical thinking dispositions were viewed by the participants seemed to be negatively influenced.

Participants who considered critical thinking dispositions to be important cited positive aspects related to personal attitudes, emotional factors, planning, personal ability and experience, and positive learning experiences as factors affecting the application of critical thinking dispositions. The teaching and learning experiences which students are exposed to do not seem to encourage critical thinking dispositions, and teachers need to find innovative ways to encourage the development of students' critical thinking dispositions in the classroom. Students need to know the value and meaning of critical thinking dispositions, and teachers need to model strategies to enhance them in their students. Assessment rubrics and checklists should be developed to informally assess students' thinking dispositions.

DISCUSSION

Although, the findings of the research may still be relative, they have implications for teaching Mathematics as well as for the training of Mathematics teachers nationally and internationally. In order to enhance achievement in Mathematics, the researchers argue that more emphasis should be placed on the explicit development of critical thinking dispositions before students can be expected to think critically in Mathematics. Although many factors can contribute to students' performance in Mathematics, problematic elements related to deficient critical thinking dispositions should not be ignored as factors that could influence achievement in Mathematics, where the application of effective critical thinking skills are required for investigating and solving problems.

The questionnaire data indicated that all the critical thinking dispositions could benefit from greater encouragement since all the means were greater than 1. Against this background, it could be argued that many dispositions could be fragile and might hamper the conceptual understanding in Mathematics of the students who took part in the study (Mueller et al. 2011). Truth seeking appears to be regarded as the most important and inquisitiveness as the least important critical thinking disposition. If students are not predisposed to truth seeking, they will not continually evaluate new information or demonstrate willingness to alter their beliefs (McBride et al. 2002). Participants' personal attitudes, emotional factors, lack of time and planning, personal failure and inadequate teaching and learning approaches seem to be factors that influence their critical thinking dispositions.

Based on the research results, the researchers deduce that it is reasonable to assume that the participants had little exposure to teaching and learning approaches that encourage the development and application of critical thinking dispositions. It appears that transmission-oriented teaching that focuses on factual information is favoured above discovery-oriented and connectionist-oriented teaching where Mathematics is regarded as a set of knowledge and intertwined concepts that is learned through student-guided explorations (Polly et al. 2013). This finding supports the view of Authors (2008) with respect to the nature of teaching and learning in South African classrooms that, despite a learner-centred curriculum, teachers do not infuse critical thinking in their daily lessons. The role of the teacher in encouraging critical thinking dispositions through a rich and participatory environment that nurtures thinking, as pointed out by Feuerstein et al. (2002), Fraser (2006), the National Center for Education Evaluation and Regional Assistance (2013) and Ottmar et al. (2014), was apparently not a part of the teaching and learning experiences of the study participants. It is therefore reasonable to assume that the study participants had not been exposed to quality instruction time during which they could develop and apply skills and dispositions (Na-

tional Center for Education Evaluation and Regional Assistance 2013; Ottmar et al. 2014). In addition, activity-based learning, learning where students are encouraged to develop the disposition to ask questions and to acquire complete information before they make final decisions (Salami 2014), is required to enhance the learning of Mathematics.

Based on the data, the researchers conclude that the desired outcomes set by the Department of Basic Education (2011), such as a systematic approach, working with others to enhance receptivity to ideas and drawing on critical consideration (Jacobs 2004; Department of Basic Education 2011), probably have not been achieved in teaching at school or tertiary level. Linked to this, the researchers argue that performance and significant outcomes in Mathematics, such as logical and critical thinking, a systematic approach and the ability to solve problems in Mathematics (Consiglio 2003; Maree et al. 2004; Winch 2006; Ayalon and Even 2008; Sezer 2008; Van de Walle et al. 2010),are likely to be problematic to students. According to Grotzer (n. d.) and Heddens et al. (2009), critical thinking dispositions are important to achieve the above outcome. If the ideals of the new curriculum and assessment policy are to be achieved in Mathematics (Department of Basic Education 2011), real actions are needed to encourage the development of critical thinking dispositions among students at school and among prospective teachers.

The data may also indicate that teachers and lecturers involved in the training of Mathematics teachers may have limited experience in encouraging critical thinking dispositions (Burns 2009). Based on the foregoing, the researchers argue that students and prospective teachers might not become ideal critical thinkers if critical thinking dispositions are not purposefully encouraged (Facione 1990, 2010).

Furthermore, the data also point to the fact that teachers and lecturers involved in teacher education are probably still focused too much on the transfer of knowledge (Dewey, quoted by Ritchhart 2002; Polly et al. 2013) and a more dynamic, problem-based and discovery-oriented way of instruction at school and tertiary level is required (Facione 2000; Polly et al. 2013; Profetto-McGrath 2003; Temel 2014). The findings support the view of McBride et al. (2002) with respect to placing emphasis on bringing students into a culture that promotes inquiry, good questions and best knowledge before decision-making. In this regard, Bonotto (2013) and Firmender et al. (2014) mention the importance of discussion to facilitate the development of critical thinking and verbal mathematical communication to enhance achievement and understanding.

The researchers are concerned that education policy particularly in South Africa and probably elsewhere, currently does not emphasize the deliberate achievement of learning outcomes related to the mastery of critical thinking dispositions, and that most participants apparently coincidentally learn the skills at home or in everyday dealings with friends.

The data support the literature regarding the fact that emotional factors, the teacher and the classroom situation can have an impact on students' critical thinking dispositions, as indicated by the participants (Carrol 2004; Gresalfi and Cobb 2006; Costa and Kallick 2009b). Time, lack of planning, encouragement, motivation and perseverance are also highlighted as factors that could affect critical thinking dispositions. These findings support the research done by Bonotto (2013), Mueller et al. (2011) and Tsanwani et al. (2014),which indicated that supportive learning environments where learners' efforts are appreciated and the teacher's questioning, as well as well-designed, appropriate and engaging tasks influence the establishment of critical thinking dispositions in Mathematics.

The international studies conducted by Facione et al. (1995), Bers et al. (1996) and McBride et al. (2002) revealed similar trends in the profiles of the students' critical thinking dispositions. In all three studies, the students appeared to be opposed to truth seeking. The studies of Facione et al. (1995) and Bers et al. (1996) revealed no difference between the students' scores for truth seeking and systematicity, but the private university students, who are regarded as stronger academically than the community college students (McBride et al. 2002),more positively endorsed inquisitiveness and open-mindedness.

Although the international studies by Facione et al. (1995), Bers et al. (1996) and McBride et al. (2002) represented an important first step in generating new knowledge and theory about the critical thinking dispositions of future teachers, the studies are limited and do not focus on Mathematics students. The training of Mathe-

matics teachers need to become a focus of interest, as accomplishment in Mathematics is important for the economic wellbeing of all countries. The present paper represents first steps in generating new knowledge and theory about the critical thinking dispositions of future Mathematics teachers in South Africa, which appear to be more fragile than the positive disposition profile of teachers constructed by McBride et al. (2002). In line with the findings of the study by Bers et al. (1996), inquisitiveness and open-mindedness are two dispositions that also appear to be fragile among the South African pre-service teachers who took part in the research. The present study could open on-going research and discussions on novel ways to gear teacher training internationally and in South Africa to focus on nurturing fragile critical thinking dispositions among Mathematics students.

CONCLUSION

The researchers acknowledge that their research is still open to further scrutiny and that no generalizations beyond the scope of the data can be made. However, the present study can serve as a point of departure for further studies to identify similarities and differences within other student populations in other contexts.

RECOMMENDATIONS

The present paper has some important implications for teacher training at national and international level. Bearing in mind the poor achievement in Mathematics at school level, one may not assume that prospective Mathematics teachers will naturally be critical thinkers who will be successful at investigating and solving problems, and capable of advancing Mathematics achievement among learners at school. It is important to establish firstly whether prospective Mathematics teachers endorse critical thinking dispositions as important, after which explicit and purposeful attempts should be made at nurturing the deficient or fragile dispositions, which could in turn possibly enhance more effective critical thinking. Since the classroom is the most obvious place where critical thinking dispositions can be nurtured, Mathematics teachers should do so explicitly during teaching. The critical thinking dispositions of prospective Mathematics teachers should be improved before they are expected to develop the critical thinking dispositions of learners at school.

The researchers explained that a social constructivist and humanistic approach to learning should characterise the teaching and learning of Mathematics in order to nurture critical thinking dispositions. A social constructivist approach to learning will allow opportunities for social learning in which dispositions such as truth seeking, open-mindedness and self-confidence in reasoning can be encouraged. In a social constructivist approach students will be expected to weigh the different perceptions and motivations of others against their own perceptions and motivations. On the other hand, a humanistic approach to learning argues that emotional aspects such as dispositions can either inhibit or facilitate learning.

Teacher training needs to be geared towards investigating ways to nurture fragile critical thinking dispositions among students in order to produce Mathematics teachers who themselves are capable of skilfully and mindfully applying cognitive tools when confronted with problems.

LIMITATIONS

The sample size of this study was very small and restricted to one university and one subject area at tertiary level, and therefore, the results cannot be generalized. Another qualitative component, such as interviews or focus group interviews, could be used to illuminate underlying factors that might affect critical thinking dispositions.

REFERENCES

Alphonso C 2013. Canada's Fall in Math-Education Ranking Sets off Alarm Bells. The Globeand Mail. From <http://www. theglobeandmail. com/news/national/educationcanadas-fall-in-math-education-ranking-sets-off-red-flags/article15730663> (Retrieved on 14 May 2013).

Anderson J 2010. *Succeeding with Habits of Mind*. Victoria, Australia: Hawker Brownlow Education.

Ayalon M, Even R 2008. Deductive reasoning: In the eye of the beholder. *AJESMS*, 69: 235-247.

Bailin S, Case R, Coombs JR, Daniels LB1999. Conceptualizing critical thinking. *JCS*, 31(3): 285-302.

Ben-Hur M 2006. *Concept-rich Mathematics Instruction: Building a Strong Foundation for Reasoning and Problem-Solving*. Alexandria, VA: Association for Supervision and Curriculum Development.

Bennie K 2005. The MATH taxonomy as a tool for analyzing course material in mathematics: A study of its usefulness and its potential as a tool for curriculum development. *AJRMSTE,* 9(2): 81-95.

Bers T, McGowan M, Rubin A1996. The disposition to think critically among community college students: The California critical thinking dispositions inventory. *J Gen Educ,* 45:197-223.

Bold C 2012. *Using Narrative in Research*. Los Angeles, CA: SAGE.

Bonotto C 2013. Artifacts as sources for problem-posing activities. *Educ Stud Math*, 83: 37-55.

Borich GD 2004. *Effective Teaching Methods*. 5th Edtion. Upper Saddle River, NJ: Pearson/Merrill Prentice Hall.

Burns E 2009. *The Use of Science Inquiry and its Effect on Critical Thinking Skills and Dispositions in Third Grade Students*. PhD Thesis, Unpublished. Chicago, IL: Loyola University.

Carroll RT 2004. Chapter One: Becoming a Critical Thinker. From <http://www. skepdic. com/refuge/ctlessons/ch1.pdf> (Retrieved on 5 December 2013).

Consiglio V 2003. How Do Students Learn Best? From <http://www6.district125.k12.il.us/~vconsigl/teacherpage/ResearchPaper.pdf> (Retrieved on 5 May 2013).

Costa AL 2009. Describing the habits of mind. In: AL Costa, B Kallick (Eds.): *Learning and Leading with Habits of Mind. 16 Essential Characteristics for Success*. Victoria, Australia: Hawker Brownlow, pp. 15-41.

Costa AL, Kallick B 2009a. Changing perspectives about intelligence. In: AL Costa, B Kallick (Eds.): *Learning and Leading with Habits of Mind. 16 Essential Characteristics for Success.*Victoria, Australia: Hawker Brownlow Education, pp. 5-14.

Costa AL, Kallick B 2009b. Creating "thought-full" environments. In: AL Costa, B Kallick (Eds.): *Learning and Leading with Habits of Mind. 16 Essential Characteristics for Success*. Victoria, Australia: Hawker Brownlow Education, pp. 97-116.

Costa AL, Kallick B 2009c. Toward a mindful language of learning. In: AL Costa, B Kallick (Eds.): *Learning and Leading with Habits of Mind. 16 Essential Characteristics for Success*. Victoria, Australia: Hawker Brownlow Education, pp. 117-134.

Creswell JW 2009. *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. 3rd Edition. Los Angeles, CA: SAGE.

Cuoco A, Goldenberg EP, Mark J 1996. Habits of mind: An organizing principle of Mathematics curricula. *J Math Behav*, 15: 375-402.

Department of Education (South Africa) 2003. *National Curriculum Statement Grades 10–12 (General) Mathematics.* Pretoria: Government Printer.

Department of Basic Education (South Africa) 2011. *Curriculum and Assessment Policy Statement: Grades 10–12 Mathematics.* Cape Town: Government Printing Works.

Department of Basic Education (South Africa) 2013. Report on the Annual National Assessment of 2013. Grades 1 to 6 and 9. From <http://www. education. gov. za/LinkClick. aspx?fileticket= Aiw7H W8ccic% 3D and tabid=36> (Retrieved on 22 November 2013).

Eggen P, Kauchak D 2004. *Educational Psychology: Windows on Classrooms*. Upper Saddle River, NJ: Pearson/Merrill/Prentice Hall.

Eggen P, Kauchak D 2010. *Educational Psychology: Windows on Classrooms*. 8th Edition. Upper Saddle River, NJ: Pearson/Merrill/Prentice Hall.

Elder L 2007. A Professional Development Model for K-12 Schools: Critical Thinking as the Key to Substantive Learning. From <http://www. criticalthinking. org/professionalDev/model-for-k-12. cfm> (Retrieved on 20 July 2013).

Elder L, Paul R 2002. Critical thinking: Distinguishing between inferences and assumptions. *J Dev Educ,* 25(3): 34-35.

Elder L, Paul R 2004. *Thinker's Guide Series – Critical Thinking Concepts and Tools.* Tomales, CA: Foundation for Critical Thinking.

Eli JA, Mohr-Schroeder MJ n. d. Mathematical connections and their relationship to mathematics knowledge for teaching geometry. *SSMA*, 113(3): 120-134.

Elyousif YAK, Abdelhamied NE 2013. Assessing secondary school teachers' performance in developing habits of mind for students. *Int Interd J Educ*, 2(2): 168-180.

English LD 2008. Interdisciplinary Problem-solving: A Focus on Engineering Experiences. Proceedings, 31st Annual Conference of the Mathematics Education Research Group of Australasia. From <http://www. merga. net. au/documents/RP192008. pdf> (Retrieved on 7 January 2013).

Facione PA 1990. *Critical Thinking: A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction. Executive Summary. "The Delphi Report*." Millbrae, CA: The California Academic Press.

Facione PA 2000. The disposition toward critical thinking: Its character, measurement, and relationship to critical thinking skill. *Informal Logic*, 20(1): 61-84.

Facione PA 2010. Critical thinking: What it is and why it counts. *Update*, 1-24.

Facione PA 2011. *Think Critically.* Boston, MA: Prentice Hall.

Facione NC, Facione PA 2010. *CCTDI Test Manual*. Millbrae, CA: California Academic Press.

Facione PA, Sanchez C, Facione N, Gainen J1995. The disposition toward critical thinking. *J Gen Educ*, 44: 1-15.

Feuerstein R 2007. The Feuerstein Instrumental Enrichment Programme (FIE). Instrumental Materials: Level 1 Training. Israel: ICELP. *Workshop presented by Lilian Lomofsky*, June 23-27, 2008. Vanderbijlpark, South Africa, North-West University.

Feuerstein R, Feuerstein RS, Falik LH, Rand Y 2002. *The Dynamic Assessment of Cognitive Modifiability.* Israel: Institute for the Enhancement of Learning Potential (ICELP).

Firmender JM, Gavin MK, McCoach DB 2014. Examining the relationship between teachers' instructional practices and students' mathematics achievement. *J Adv Acad*, 25(3): 214-236.

Fradkin L, Zernov V, Crisan C, Lerman S 2010. First year engineering mathematics: The London South Bank University experience. *MSOR Connections*, 10(1): 35-39.

Francisco JM 2013. The mathematical beliefs and behavior of high school students: Insights from a longitudinal study. *J Math Behav*, 32:481-493.

Fraser JDC 2006. Mediation of learning. In: MM Nieman, RB Monyai (Eds.): *The Educator as Mediator of Learning.* Pretoria: Van Schaik, pp. 1-21.

Fredericks I 2012. Annual Tests Sum Up A Sorry State. From <http://m.iol.co.za/article/view/s/81/a/333812> (Retrieved on 10 December 2012).

Grabe M, Grabe C 2004. *Integrating Technology for Meaningful Learning.* 4th Edition. New York: Houghton Mifflin.

Gresalfi MS, Cobb P 2006. Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies*, 1(1): 49-57.

Grotzer TA n. d. Learning the Habits of Mind That Enable Mathematical and Scientific Behavior. Math/Science Matters: Resource Booklets on Research in Math and Science Learning. Booklet 2. Issues Of Instructional Technique in Math and Science Learning. From <http://www. learnweb. harvard. edu/alps/thinking/docs/habits. doc> (Retrieved on 2 June 2013).

Gunhan BC 2014. A case study on the investigation of reasoning skills in geometry. *SAJE*, 34(2): 1-19.

Hanford E 2011. Don't Lecture Me. Rethinking the Way College Students are Taught. From <http://www. americanradioworks. publicradio. org/features/tomorrows-college/lectures/rethinking-teaching. html> (Retrieved on 23 January 2013).

Heddens JW, Speer WR, Brahier DJ 2009. *Today's Mathematics: Concepts, Methods, and Classroom Activities.* Hoboken, NJ: Wiley.

Jacobs M 2004. Outcomes. In: M Jacobs, NCG Vakalisa, N Gawe (Eds.): *Teaching-learning Dynamics: A Participative Approach for OBE*. 3rd Edition . Sandton, South Africa: Heinemann, pp. 87-116.

Johnson SK 2012. Rethinking the Way Colleges Teach Critical Thinking. SciAm. From <http://www. blogs. scientificamerican.com/guest-blog/2012/12/14/rethinking-the-way-colleges-teach-critical-thinking> (Retrieved on 23 August 2013).

Kilpatrick J 2011. Review of Math Performance in Global Perspective. From <http://www. nepo. coloradoedu/files/TTR-Gloval-math-O.pdf> (Retrieved on 4 June 2013).

Krulik S, Rudnick JA 1999. Innovative tasks to improve critical- and creative-thinking skills. In: LV Stiff, FR Curcio (Eds.): *Developing Mathematical Reasoning in Grades K –12, 1999 Yearbook.* Reston, VA: National Council of Teachers of Mathematics, pp. 138-145.

Leong YH, Yap SF, Quek KS, Tay EG, Tong CL, Ong YT, Chia ASFS, Zaini IKM, Khong WC, Lock OL, Zhang QTB, Tham YHT, Noorhazman NNM 2013. Encouraging problem-solving dispositions in a Singapore classroom. *Int J Math Educ Sci Tech*, 44(8): 1257-1273.

Lombard K, Grosser M 2008. Critical thinking: Are the ideals of OBE failing us or are we failing the ideals of OBE? *South African Journal of Education*, 28(4): 561–579.

M·rcuþ I 2005. Critical thinking – Applied to the methodology of teaching mathematics. *Educaþia Mathematic·*, 1(1): 57-66.

Maree JG, Louw CJ, Millard S 2004. Die impak van tutoriale op die wiskundeprestasie van eerstejaarstudente (The impact of tutorials on the Mathematics performance of first-year students, in Afrikaans). *Tydskrifvir Natuurwetenskap en Tegnologie*, 23(1): 25-34.

Matsuura R, Sword S, Piecham MB, Stevens G, Cuoco A 2013. Mathematical habits of mind for teaching: Using language in Algebra classrooms. *The Mathematics Enthusiast*, 10(3): 735-776.

McBride RE, Xiang P, Wittenburg D 2002. Dispositions toward critical thinking: The pre-service teachers' perspective. *Teach Teach Educ*, 8(1): 29-40.

McMillan JH, Schumacher S 2006. *Research in Education: Evidence-based Inquiry.* 6th Edition. Boston, MD: Allyn and Bacon.

Mueller M, Yankelewitz D, Maher C 2011. Sense making as motivation in doing Mathematics: Results from two studies. *The Mathematics Educator*, 20(2): 33-43.

NCTM 1989. National Council of Teachers of Mathematics. Evaluation: Standard 10–Mathematical Disposition. From <http://www.usi.edu/science/math/sallyk/Standards/Previous/CurrEvStds/evals10. htm> (Retrieved on 15 September 2012).

National Center for Education Evaluation and Regional Assistance 2013. *Instructional Practices and Student Math Achievement: Correlations from a Study of Math Curricula.* Washington DC.

Nieuwenhuis J 2007. Analysing quantitative data. In: K Maree (Ed.): *First Steps in Research.* Pretoria: Van Schaik, pp. 98-122.

Nisbett RE, Norenzayan A 2002. Culture and cognition. In: DL Medin (Ed.): *Stevens' Handbook of Experimental Psychology.* 3rd Edition. New York: Wiley, pp. 3-28.

Nisbett RE, Peng K, Choi I, Norenzayan A 2001. Culture and systems of thought: Holistic versus analytic cognition. *Psychol Rev*, 108(2): 291-310.

O'Daffer P, Charles R, Cooney T, Dossey J, Schielack J 2008. *Mathematics for Elementary School Teachers.* 4th Edition Boston, MD: Pearson.

Ottmar ER, Decker LE, Cameron CE, Curb TW, Rimm-Kaufman SE 2014. Classroom instructional quality, exposure to mathematics instruction and mathematics achievement in fifth grade. *Learning Environ Res*, 17: 243-262.

Palmer SM 2007. *Critical Thinking Dispositions of Part-Time Faculty Members Teaching at the College Level.* PhD Thesis, Unpublished. Athens, OH: Ohio University.

Paul R, Elder L 2008. The Miniature Guide to Critical Thinking. Concepts and Tools. From <http://www. vault.hanover. edu/~ahrens/CT.pdf.pdf> (Retrieved on 12 November 2013).

Polly D, McGee JR, Wang C, Lambert RG, Pugalee DK, Johnson S 2013. The association between teachers' beliefs, enacted practices, and student learning in Mathematics. *The Mathematics Educator*, 22(2): 11-30.

Profetto-McGrath J 2003. The relationship of critical thinking skills and critical thinking dispositions of baccalaureate nursing students. *J Adv Nurs*, 43(6): 569-577.

Qing Z, Ni S, Hong T 2010. Developing critical thinking dispositions by task-based learning in chemistry experiment teaching. *Procedia–Soc Behav Sci*, 2: 4561-4570.

Ricketts JC, Irani T, Jones L 2003. The effect of instructional delivery methods on the critical thinking disposition of distance students and traditional on-campus students. *J S Agric Educ Res*, 53(1): 59-71.

Ritchhart R 2002. *Intellectual Character: What It Is, and Why It Matters, and How to Get It.* San Francisco, CA: Jossey-Bass.

Salami, IA 2014. Hands-on/Mind-on activity-based strategy: The effect on pre-service teachers' subject matter knowledge in a primary mathematics course. *J Emerging Trends Educ Res Policy Stud*, 5(7): 96-103.

Saleh IM, Khine MS 2011. *Fostering Scientific Habits of Mind: Pedagogical Knowledge and Best Practices in Science Education.* Rotterdam, Netherlands: Sense Publishers.

Schoenfeld AH 1992. Learning to think mathematically: Problem-solving, metacognition, and sense-making in mathematics. In: DA Grouws (Ed.): *Handbook for Research on Mathematics Teaching and Learning*. New York: Macmillan, pp. 334-370.

Sezer R 2008. Integration of critical thinking skills into elementary school teacher education courses in Mathematics. *J Educ*, 128(3): 349-362.

Staples M 2007. Supporting Whole-Class Collaborative Inquiry in a Secondary Mathematics Classroom. Center for Research in Mathematics Education Publications, Paper 1. From <http://www. digitalcommons.uconn. edu/merg_docs/1> (Retrieved on 5 July 2013).

Temel S 2014. The effects of problem-based learning on pre-service teachers' critical thinking dispositions and perceptions of problem-solving ability. *S Afr J Educ*, 34(1): 1-20.

Tileston DW 2005. *10 Best Teaching Practices: How Brain Research, Learning Styles, and Standards Define Teaching Competencies.* 2nd Edition. Thousand Oaks, CA: Corwin Press.

Tsanwani A, Harding A, Engelbrecht J, Maree K 2014. Perceptions of teachers and learners about factors that facilitate learners' performance in Mathematics in South Africa. *Afr J Res Math, Sci Technol Educ*, 18(1): 40-51.

Van de Walle JA 2004. *Elementary and Middle School Mathematics: Teaching Developmentally.* 5th Edition. New York: Allyn and Bacon.

Van de Walle JA, Karp KS, Bay-Williams FM 2010. *Elementary and Middle School Mathematics: Teaching Developmentally.* 7th Edition. New York: Allyn and Bacon.

Weimer M 2010. To Improve Students' Problem-Solving Skills Add Group Work to the Equation. From <http://www. facultyfocus. com/articles/teaching-and-learning/to-improve-students-problem-solving-skills-add-group-work-to-the-equation/> (Retrieved on 25 August 2013).

Winch C 2006. *Education, Autonomy and Thinking*. New York: Routledge.

Woolfolk A 2004. *Educational Psychology.* 9th Edition. Cape Town: Pearson.